§14.6

<u>Note</u>: Given $f(x,y,z)$ & $P(x_0,y_0,z_0)$ on S_k.

then $\vec{\nabla} f(P) \perp$ the tangent plane to S_k through P.

when $\vec{\nabla} f(P) \neq \vec{0}$.

- If $M(x,y,z)$ is a pt on the tangent plane then $\vec{\nabla} f(P) \perp \vec{PM}$ (i.e. $\vec{\nabla} f(P) \cdot \vec{PM} = 0$)

- If $M(x,y,z)$ on the normal line, then $\vec{PM} \parallel \vec{\nabla} f(P)$ (i.e. $\vec{PM} = t\,\vec{\nabla} f(P)$; $t \in \mathbb{R}$).

Remark : If $z = f(x,y)$; $P(x_0, y_0, z_0)$ on graph(f)

for example $z = x^2 + y^2 - 2x + 1$

Create a new function : $g(x,y,z) = x^2 + y^2 - 2x + 1 - z$

See that graph(f) $\equiv S_0$ for g.

$$\boxed{z_0 = f(x_0, y_0)}$$

5) $\cos \pi x - x^2 y + e^{xz} + yz = 4$; $P(0,1,2)$

$\overrightarrow{PM} = \begin{vmatrix} x \\ y-1 \\ z-2 \end{vmatrix}$

a) let $f(x,y,z) = \cos \pi x - x^2 y + e^{xz} + yz$

$\overrightarrow{\nabla f} = (-\pi \sin \pi x - 2xy + ze^{xz})\vec{i} + (z - x^2)\vec{j} + (xe^{xz} + y)\vec{k}$

$\overrightarrow{\nabla f}(P) = 2\vec{i} + 2\vec{j} + \vec{k}$

let $M(x,y,z)$ be a pt on the tangent plane, then $\overrightarrow{PM} \cdot \overrightarrow{\nabla f}(P) = 0$

$\therefore 2x + 2(y-1) + z - 2 = 0 \Longrightarrow 2x + 2y + z = 4$

b) let $N(x,y,z)$ be a pt on the normal line, then $\overrightarrow{PN} = t\,\overrightarrow{\nabla f}(P)$ $t \in \mathbb{R}$.

$\therefore \begin{cases} x = 2t \\ y - 1 = 2t \\ z - 2 = t \\ t \in \mathbb{R} \end{cases} \Longrightarrow \begin{cases} x = 2t \\ y = 1 + 2t \\ z = 2 + t \\ t \in \mathbb{R} \end{cases}$

tangent line to the

Curve of intersection of two surfaces (at a pt P)

$f(x,y,z) \longrightarrow S_{k_1}$

; $C = S_{k_1} \cap S_{k_2}$

$g(x,y,z) \longrightarrow S_{k_2}$

$P(x_0,y_0,z_0) \in C$

let $M(x,y,z)$ be a pt on the tangent line

$$\overrightarrow{PM} = t\left(\overrightarrow{\nabla f} \times \overrightarrow{\nabla g}\right)$$

$t \in \mathbb{R}$.

14

$xyz=1$

$x^2+2y^2+3z^2=6$

$P(1,1,1)$

let $f(x,y,z)=xyz$

& $g(x,y,z)=x^2+2y^2+3z^2$

$\vec{\nabla} f = yz\vec{i}+xz\vec{j}+xy\vec{k}$

$\vec{\nabla} f(P)=\vec{i}+\vec{j}+\vec{k}$

$\vec{\nabla} g = 2x\vec{i}+4y\vec{j}+6z\vec{k}$

$\vec{\nabla} g(P)=2\vec{i}+4\vec{j}+6\vec{k}$

$$\vec{w}=\vec{\nabla} f(P)\times\vec{\nabla} g(P)=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 1 & 1\\ 2 & 4 & 6\end{vmatrix}$$

$=2\vec{i}-4\vec{j}+2\vec{k}$

let $M(x,y,z)$ be a pt on the tangent line.

$\vec{PM}=\begin{vmatrix}x-1\\ y-1\\ z-1\end{vmatrix}$

$\vec{PM}\parallel\vec{w}$ $\boxed{\vec{PM}=t\vec{w}}$

$\therefore$ $x-1=2t$

$y-1=-4t$

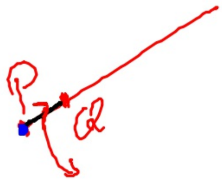
$z-1=2t$

$t\in\mathbb{R}$.

...

Given $f(x,y)$ or $f(x,y,z)$

$$\boxed{\text{Estimating the change in } f \text{ in a given direction } \vec{u}. \quad df \cong D_u f(P)\, ds}$$



$$\vec{u} = \frac{\vec{PQ}}{|\vec{PQ}|}$$

$f(x,y)$

$$df = f_x\, dx + f_y\, dy$$

Recall

$y = f(x)$

$dy = df = f'(x)\, dx$

$\nabla f = f_x \vec{i} + f_y \vec{j}$

$\vec{u} = \vec{i}$

$D_u f = f_x$

20

$f(x,y,z) = e^x \cos yz$

$\vec{v} = 2\vec{i} + 2\vec{j} - 2k.$ $\longrightarrow \vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{1}{2\sqrt{3}}(2\vec{i} + 2\vec{j} - 2\vec{k})$

$P_0(0,0,0)$; $ds = 0.1.$

$\vec{u} = \frac{\vec{i}}{\sqrt{3}} + \frac{\vec{j}}{\sqrt{3}} - \frac{\vec{k}}{\sqrt{3}}$

$\vec{\nabla} f = e^x \cos yz \, \vec{i} - z e^x \sin yz \, \vec{j} - y e^x \sin yz \, \vec{k}$

$\vec{\nabla} f(0,0,0) = \vec{i}$

$df \simeq (D_u f)\, ds =$

$= \left((\vec{i}) \cdot \left(\frac{\vec{i}}{\sqrt{3}} + \frac{\vec{j}}{\sqrt{3}} - \frac{\vec{k}}{\sqrt{3}} \right) \right) (0.1) = \frac{0.1}{\sqrt{3}}.$

21 (same as #20)

$g(x,y,z) = x + x\cos z - y\sin z + y$

$P_0(2,-1,0)$; $P_1(0,1,2)$; $ds = 0.2$

$\vec{\nabla g} = (1+\cos z)\vec{i} + (1-\sin z)\vec{j} + (-x\sin z - y\cos z)\vec{k}$

$\vec{\nabla g}(P_0) = 2\vec{i} + \vec{j} + \vec{k}$

$dg \simeq (D_u f)\, ds = 0$

$\vec{P_0P_1} = -2\vec{i} + 2\vec{j} + 2\vec{k}$

$\vec{u}\ \dfrac{1}{2\sqrt{3}}(-2\vec{i} + 2\vec{j} + 2\vec{k})$

$\vec{u} = \dfrac{1}{\sqrt{3}}(-\vec{i} + \vec{j} + \vec{k})$

$D_u f = -\dfrac{2}{\sqrt{3}} + \dfrac{1}{\sqrt{3}} + \dfrac{1}{\sqrt{3}} = 0$